

Figure A-1. Flow Chart for Calculating Estimates of Autocorrelation and Partial Autocorrelation Functions.

2. Preliminary Estimates of Parameters

The equations for calculating preliminary estimates of the parameters for the identified model are given in Chapter 2 [(Equations (2.38) through (2.42)] and will not be repeated here. Figure A-2 is a flow chart for implementing these equations. As shown in the flow chart, the tolerance on the f vector has been set at 0.0001. Any other value appropriate to the particular problem could be used. Table A-II contains the BASIC program listing.

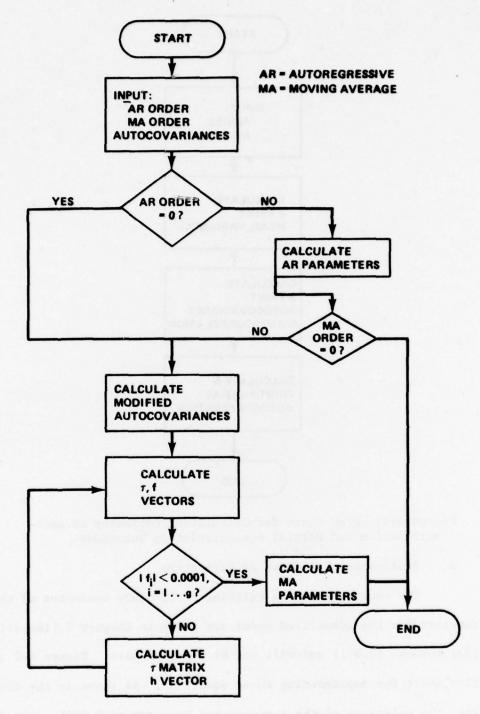


Figure A-2. Flow Chart For Calculating Preliminary Estimates of Model Parameters.

TABLE A-I. BASIC PROGRAM LISTING FOR CALCULATING ESTIMATES OF AUTOCORRELATION AND PARTIAL AUTOCORRELATION FUNCTIONS

```
10 DIM 28(2001, C[25], R[25], WS[200], PS[25, 25], A$[21], B$[21]
 15 A$=
20 DISP "ENTER DATA FILE NR";
 30 INPUT F
40 LOAD DATA F.Z
50 DISP "ENTER NR LAGS";
60 INPUT K
70 DISP "ENTER NR SAMPLES";
80 INPUT N
90 S1=0
100 S2=0
110 FOR I=1 TO N
 120 81=81+2011
 130 NEXT I
 140 M=S1/N
 150 81=0
 160 FOR 1=1 TO N
 170 WE I 3=ZE I 3-M
180 S1=S1+W[1]
190 S2=S2+W[1]*W[1]
 200 NEXT I
210 V2=S2/(N-1)
215 PRINT "MEAN=";N; "VARIANCE=";V2
 228 FOR J=1 TO K
 230 53=0
 240 FOR I=1 TO N-J
250 L=I+J
260 S3=S3+W[L]+W[I]
 270 NEXT 1
 280 C[ J]=93/N
 290 REJJ=CEJJ/V2
 300 NEXT J
310 PRINT "LAG
312 PRINT " 0 "; V2
                          ACV
 314 FOR J=1 TO K
 316 PRINT JICEJI
 318 NEXT J
320 WRITE (15,322)
322 FORMAT /, AUTOCORRELATION FUNCTION //,29X,"-1",9X,"0",9X,"1",/,30X,21"I"
 324 FOR J=1 TO K
326 B$="
328 IF R[J](0 THEN 338
 330 H=INT(10*R[J]+0.5)
 332 B$[11:11+H]=A$[11:11+N]
 336 GOTO 345
 338 N=INT(10*R[J]+0.5)
 340 B$[11+N,11]=A$[11+N,11]
 345 PRINT J:R[ J], TAB29, B$
 350 NEXT J
```

TABLE A-1. (CONCLUDED)

TABLE A-II. BASIC PROGRAM LISTING FOR CALCULATING PRELIMINARY ESTIMATES OF MODEL PARAMETERS

```
10 REM PRELIMINARY ESTIMATE OF ARMA PARAMETERS
20 DIM ASC 10, 10], BSC 10, 10], FSC 10], TSC 10], CSC 10], PSC 10], QSC 10]
40 INPUT P1.01
50 FOR J=1 TO 10
60 DISP "ENTER C("; J;")";
70 INPUT CLJ]
80 NEXT J
90 IF P1=0 THEN 450
100 REDIM ALPI.P1 1.0[P1 ].P[P1 ].B[F1.F1]
110 FOR I=1 TO P1
120 FOR J=1 TO P1
130 K=ABS(Q1+I-J)+1
140 ACT. J 3=CEK 3
150 NEXT J
160 Q( | ]=C(Q1+1+1 ]
170 NEXT I
180 IF DET(A)=0 THEN 240
190 MAT B=INV(A)
200 MAT P=B+W
210 PRINT "AUTOREGRESSIVE PARAMETERS ARE"
220 MAT PRINT P
222 S1=C[1]
223 FOR J=1 TO P1
224 S1=S1-P(JJ*C(J+1)
225 NEXT J
230 GOTO 260
240 PRINT "A NOT INVERTIBLE"
250 STOP
260 IF 01=0 THEN 770
262 REDIM QCQ1+1 1.PCP1+1 1
265 FOR J=1 TO Q1+1
275 Qt J J=0
280 FOR I=1 TO P1+1
285 D=0
290 FOR K=1 TO P1+1
300 IF K>1 THEN 320
310 X=-1
315 GOTO 330
320 X=FLK-11
330 L=ABS(J+I-K-1)+1
340 D=D+X+C[L]
350 NEXT K
360 IF I>1 THEN 380
365 Y=-1
370 GOTO 390
380 Y=P[[-1]
390 Q[J]=Q[J]+Y*D
400 NEXT I
410 NEXT
420 FOR J=1 TO Q1+1
430 C[J]=Q[J]
440 NEXT J
450 REDIM TEQ:+1 ], FEQ:+1 ], AEQ:+1, Q:+1, D:BEQ:+1, Q:+1 ], HEQ:+1 ], Q:Q:+1 ]
460 T[1]=(C[1])+0.5
470 FOR J=2 TO Q1+1
480 T[ J]=0
486 NEXT J
490 FOR J=0 TO Q1
500 D=0
```

TABLE A-II. (CONCLUDED)

```
510 FOR I=0 TO Q1-J
520 D=D+T[[+1]+T[[+J+1]
530 NEXT I
540 F[J+1]=D-C[J+1]
550 NEXT J

552 Z=0

554 FOR J=1 TO Q1+1

555 IF ABS(F[J])(0.0001 THEN 557

556 Z=Z+1

557 NEXT J
557 NEXT J

558 PRINT Z

559 IF Z=0 THEN 770

560 NAT A=ZER

570 MAT B=ZER

575 MAT FRINT F

580 FOR I=0 TO 01

590 FOR J=0 TO 01-I
600 A[ I+1.J+1 ]=T[ J+1+1 ]
610 NEXT J
620 NEXT I
630 FOR I=1 TO 01+1
640 FOR J=1 TO 01+1
650 B[ 1, J ]=T[ J-1+1 ]
660 NEXT J
670 NEXT I
680 MAT H=H+B
690 IF DET(A)=0 THEN 750
700 MAT B=INV(A)
710 MA1 Q=8*F
730 MAT T=1-0
740 GOTO 490
750 PRINT "T MATRIX NOT INVERTIBLE"
760 STOP
770 X=1
780 IF P1=0 THEN 820
790 FOR J=1 TO P1
800 X=X-P(J)
810 NEXT J
820 PRINT "OVERALL CONSTANT=W BAR TIMES";X
825 IF Q1=0 THEN 910
830 REDIM QCQ11
840 FOR J=1 TO Q1
850 Q[ J]=-T[ J+1 ]/T[ 1 ]
860 NEXT J
870 PRINT "MOVING AVERAGE PARAMETERS ARE"
880 MAT PRINT 0
890 IF Q1=0 THEN 910
900 S1=T[ 1 ]+2
910 PRINT "RESIDUAL VARIANCE IS":SI
920 END
```

Appendix B

OPTIMIZATION ALGORITHMS

Finding a maximum likelihood estimate of the parameters and starting values for the time series model requires finding the set of parameters and starting values that minimizes the sum of the squared residuals. Any stationary ARMA model can be written in the form

$$a_{t} = (w_{t} - \overline{w}) - \phi_{1} (w_{t-1} - \overline{w}) - \dots - \phi_{p} (w_{t-p} - \overline{w})$$

$$+ \theta_{1} a_{t-1} + \dots + \theta_{q} a_{t-q} , \qquad (B.1)$$

where

$$\overline{w} = \frac{1}{n} \sum_{t=1}^{n} w_{t} .$$

Given p initial values of the w's and q initial values of the a's, Equation (B.1) can be solved recursively to obtain the sequence at, from which the sum of squares,

$$s = \sum_{t=1}^{n} a_{t}^{2}$$
 (B.2)

is calculated. Letting $\underline{\xi} = (w, w_1, \dots, w_p, \phi_1, \dots, \phi_p, \theta_1, \dots, \theta_q)$ be the vector of parameters to be estimated, the vector $\underline{\xi}$ * that minimizes S can be found by any of the well-known optimization algorithms. Three algorithms, an exhaustive grid search, a linearized regression, and a pattern search, were used in this investigation, and are described here. In implementing all of these algorithms on the Hewlett-Packard 9830, the

appropriate form of Equation (B.1) was input as a defined function, and the q initial values of the residuals were set equal to zero. Also, the check for stationarity and invertibility was performed off-line.

Grid Search 1.

The grid search is a simple exhaustive search routine in which each element of the vector, &, is systematically allowed to take on each of a set of predetermined values. For instance, if $\xi_0 = (\xi_{01} \cdots \xi_{0j} \cdots \xi_{0m})$, then $\underline{\xi}$ is allowed to take on values $(\xi_{o1}...\xi_{oj}...\xi_{om})$, $(\xi_{11}...\xi_{oj}...\xi_{om})$, ..., $(\xi_{n1}...\xi_{oj}...\xi_{om})$, ..., $(\xi_{n1}...\xi_{nj}...\xi_{nm})$ until each possible combination of elements has been used. That combination which minimizes S is an estimate of the best value of t.

The exhaustive grid search is not very useful for finding an optimum value for the parameter vector, ¿, simply because it requires a large number of trials (m n where m is the dimension of $\underline{\xi}$ and n is the number of values tried for each element) to search a very large parameter space and the resolution is dependent on the interval between values of the elements. However, it is useful for mapping contours of the sum of squares function and for examining behavior in the vicinity of an estimate of the optimum point. Figure B-l is a flow chart for this algorithm; Table B-I is the computer program listing.

2. Linearized Regression

Suppose that Equation (B.1) is written as

$$a_{t} = f(\underline{t}) \tag{B.3}$$

where f is some function of the parameter vector, &. Let & be of dimension m. Expanding at in a Taylor's series about some guessed parameter vector, to, and neglecting higher order terms gives

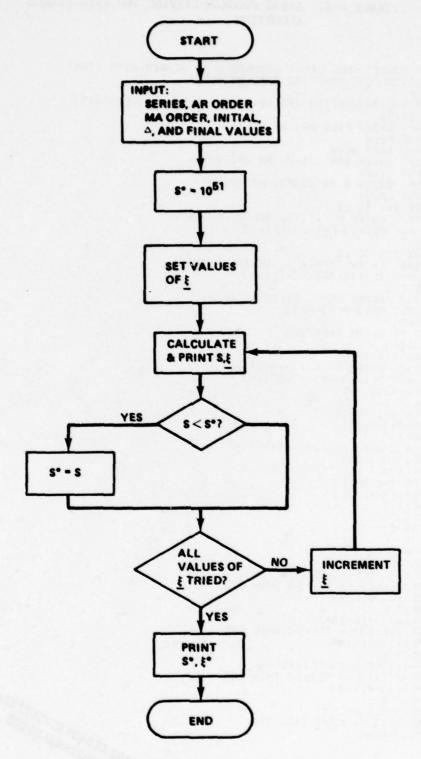


Figure B-1. Flow Chart for Grid Search Algorithm.

TABLE B-I. BASIC PROGRAM LISTING FOR GRID SEARCH ALGORITHM

```
10 REM CONDITIONAL LEAST SQUARES GRID SEARCH WITH START 15 DISP "DEFINE FUNCTION ACT" AT 5000":
15 DISP
17 STOP
20 DIM ASC 200 1. WSC 200 1. PSC 10.51. TSC 10.51. MC 51. BSC 10.51
25 S5=1E+51
30 DISP "ENTER FILE NR, NR PTS";
40 INPUT AL.N
50 LOAD
           DATA AL.W
60 DISP "ENTER #AR COEFS, AR ORDER";
70 INPUT LIPE
          "ENTER # MA COEFS. MA ORDER"!
80 DISP
90 INPUT L2:01
100 FOR 1=1 TO L1
110 DISP "ENTER P(": II"), DELTA. PSTOP";
120 INPUT PE 1.11, PE 1.21, PE 1.31
130 NEXT I
140 FOR I=1 TO L2
150 DISP "ENTER T(":I:"), DELTA, ISTOP":
160 INPUT T[[:1],T[]:2],T[]:3]
170 NEXT 1
172 DISP "ENTER MEAN, DELTA, STOP";
175 INPUT ME11, ME21, NE31
180 03=P1
190 IF P1 >= Q1 THEN 203
200 03=01
203 FOR I=1 TO 03
204 DISP "ENTER B(": 1;") DELTA, STOP";
205 INPUT BEI-11.8(1.2).8(1.3)
206 B[ 1, 4 ]=B[ 1, 1 ]
207 NEXT I
210 FOR I=1 TO L1
220 P[ 1, 4 ]=P[ 1, 1 ]
225 NEXT 1
230 FOR J=N+03 TO 03+1 STEP -1
234 W[ J ]=W[ J-03 ]
236 NENT
240 FOR I=1 TO L2
250 T[ 1, 4 ]=T[ 1, 1 ]
260 NEXT 1
262 ME 4 J=ME 1 J
270 FOR I=1 TO L1
280 FOR K=1 10 L2
285 FOR L=1 TO Q3
290 GOSUB 2000
292 B[L.1]=B[L.1]+B[L.2]
294 IF B[L.1] (= B[L.3] THEN 290
296 B[L.1]=B[L.4]
298 NEXT L
300 T[K, 1]=T[K, 1]+T[K, 2]
310 IF TEK: 11 (= TEK: 31 THEN 285
320 TEK.1 ]=TEK.41
330 NEXT K
340 P[[,1]=P[[,1]+P[[,2]
345 IF P[[,1] (= P[[,3] THEN 280
350 P[ 1, 1 ]=P[ 1, 4 ]
360 NEXT I
370 MC1 ]=MC1 ]+MC2]
380 IF ME11 <= ME31 THEN 270
390 ME11=ME4)
```

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TABLE B-I. (CONTINUED)

```
391 PRINT "BEST POINT"
392 PRINT "MERN="IMES]
394 FOR J=1 TO L1
395 PRINT "PHI("|J|")="|P[J,5]
396 NEXT J
397 FOR J=1 TO L2
398 PRINT "THETA ("; J; ")="; T[ J.5]
399 NEXT J
400 FOR J=1 TO Q3
410 PRINT "W(":J:")=":B[J.5]
420 NEXT J
430 FRINT "MIN SS=": S5: "VARIANCE=": S5 /N
432 PRINT "PRESS CONT TO RECORD RESIDUALS"
433 PRINT
435 STOP
440 MC 1 ]=ME 5 ?
450 FOR J=1 TO L1
460 P[J. 1]=P[J. 5]
470 NEXT J
480 FOR J=1 TO L2
490 T[J. | ]=T[J. 5]
500 NEXT J
510 FOR J=1 TO 03
520 B[ J. 1 ]=B[ J. 5 ]
530 NEXT J
540 GOSUB 2000
550 FOR J=1 TO N
560 A[ J]=A[ J+Q3 ]
570 NEXT J
580 DISP "INPUT FILE NR FOR RESIDUALS";
590 INPUT A2
600 STORE DATH A2.A
610 STOP
2000 FOR J=1 TO 03
2005 W(J)=B(J,1]
2007 PRINT "W(";J;")=";W(J)
2010 A(J)=0
2020 NEXT J
2030 81=0
2040 FOR J=03+1 TO N+03
2050 AL J 1= FNA (J)
2060 SI=SI+AC J]+2
2070 NEXT J
2080 S2=S1/N
2085 PRINT "MEAN=";M[1]
2090 FOR J=1 TO L1
2100 PRINT "PHI("; J; ")=";P[J:1]
2110 NEXT J
2120 FOR J=1 TO L2
2130 PRINT "THETA ("!J!")="!TEJ:11
2140 NEXT J
2142 IF S1 >= S5 THEN 2154
2143 MC5 J=NC1 J
2144 FOR J=1 TO L1
2145 P[J.5]=F[J.1]
2146 NEXT J
2147 FOR J=1 TO L2
2148 Ti J.5 J=TI J.11
```

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TABLE B-I. (CONCLUDED)

```
2149 NEXT J
2150 FOR J=1 TO Q3
2151 B[J,5]=B[J,1]
2152 NEXT J
2153 S5=S1
2154 PRINT "SUM SQUARES=";S1; "VARIANCE=";S2
2155 PRINT
2160 RETURN
2170 END
5000 DEF FNA(T)
5010 G=W[T]-P[1,1]+W[T-1]-P[2,1]+W[T-2]-M[1]+(1-P[1,1]-P[2,1])+T[1,1]+A[T-1]
5020 RETURN G
```



$$a_t = a_{t,o} - \sum_{i=1}^{m} (\xi_i - \xi_o) x_{i,t}$$
 (B.4)

where $a_{t,o} = a_t$ given the sequence $(w_1 - \overline{w}), \dots (w_n - \overline{w})$ and $\underline{\xi} = \underline{\xi}_0$, while

$$x_{i,t} = -\frac{\partial a_t}{\partial \xi_i}\Big|_{\underline{\xi} = \underline{\xi}_0}$$

Rearranging Equation (B.4),

$$a_{t,o} = \sum_{i=1}^{m} (\xi_i - \xi_o) x_{i,t} + a_t$$
 (B.5)

The set of Equations (B.5) at each value of t, t = 1, 2, ... n, can be written as the matrix equation

$$\underline{\underline{a}}_{0} = \underline{x} \left(\underline{\xi} - \underline{\xi}_{0}\right) + \underline{a}_{\underline{\xi}} \tag{B.6}$$

where \underline{x} is the nxm matrix $(x_{i,t})$ and \underline{a}_{o} and \underline{a}_{t} are column vectors with n elements. Equation (B.6) is in the proper form for a linear regression of the a_{o} 's onto the x's; therefore, the value of the vector $(\underline{t} - \underline{t}_{o})$ which minimizes

$$S = \sum_{t=1}^{n} a_t^2 = \underline{a}^T \underline{a}$$

can be found by the well-known solution of the normal equations for linear least squares regression. Thus, a minimum occurs when

$$(\underline{\xi} - \underline{\xi}) = (\underline{x}^{\mathrm{T}}\underline{x})^{-1} \underline{x}^{\mathrm{T}}\underline{a}_{0} \qquad (B.7)$$

In general, the $a_{\underline{t}}$'s are not strictly linear in the parameters, $\underline{\xi}$. Hence, a single calculation of $(\underline{\xi} - \underline{\xi}_0)$ will not produce a minimum sum of the squared residuals. Repeated application of the process with the

new value of <u>t</u> substituted for <u>t</u> will usually result in convergence if the initial estimates of the parameters are reasonably good guesses. Caution must be exercised when using this algorithm when the process is close to nonstationarity or noninvertibility, because the indicated optimum can easily be in a region of infeasibility. While the partial derivatives may be obtained directly, it is usually much easier to obtain numerical estimates to sufficient accuracy using the relationship

$$x_{i,t} = \{(a_t | (\underline{w} - \overline{w}), \xi_{1,0} \dots \xi_{i,0} \dots \xi_{m,0}) - (a_t | (\underline{w} - \overline{w}), \xi_{1,0} \dots \xi_{i,0} + \delta_{i} \dots \xi_{m,0})\} / \delta_i , \quad (B.8)$$

where (w - w) is the vector consisting of elements of the time series.

In implementing this algorithm, the value of the sum of squared residuals, $S(\xi)$, is calculated for each solution of the parameter vector, $\underline{\xi}$. If $S(\underline{\xi}) > S(\underline{\xi}_0)$, the vector difference, $\underline{\xi} - \underline{\xi}_0$, is halved and $S(\xi)$ is calculated at the new estimate of $\underline{\xi}$. This process is repeated until some value $S(\underline{\xi}) < S(\underline{\xi}_0)$ or until the absolute value of each element of the vector difference, $(\underline{\xi} - \underline{\xi}_0)$, is less than the corresponding element of some tolerance vector, $\underline{\xi}$, in which case convergence is assumed to have occurred. Figure B-2 is a flow chart of the linearized regression algorithm; Table B-II is the program listing.

3. Pattern Search

The pattern search is a heuristic algorithm that has the advantages of accelerating and following ridges. It works as follows. The inital estimate of the parameter vector, \(\frac{\xi}{\xi}\), is designated as a

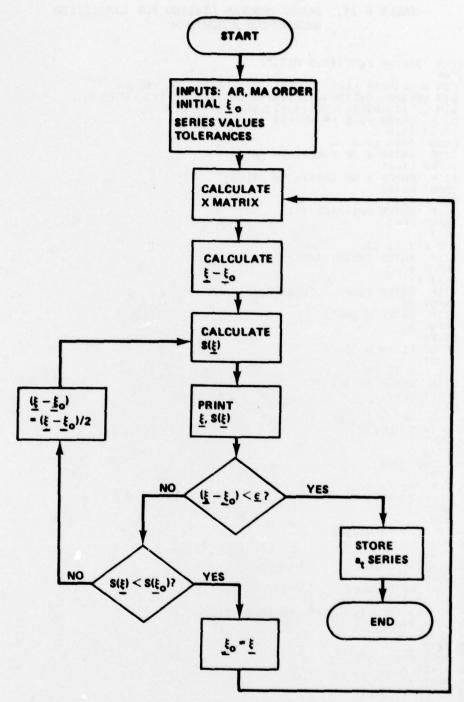


Figure B-2. Flow Chart For Linearized Regression Algorithm.

TABLE B-II. BASIC PROGRAM LISTING FOR LINEARIZED REGRESSION ALGORITHM

```
1 DISP "DEFINE FUNCTIONS ACTO";
9 STOP
10 PEM NONLINEAR LEAST SQUARES WITH STARTING VALUES 20 DIM WSC 200], ASC 150], PSC 10], TSC 10], MSC 150, 11], YSC 11, 11] 30 DIM GSC 11], HSC 11], USC 11, 11], BSC 10], DSC 11]
40 DISP "ENTER FILE NR. NR PTS";
50 INPUT ALAN
51 LOAD DATH AL.W
60 DISP "ENTER # AR COEFS, AR ORDER";
61 INPUT L1,P1
62 DISP "ENTER # MA COEFS, MA ORDER";
63 INPUT L2,01
64 FOR K=1 TO L1
65 DISP "ENTER PHI("IK;")";
66 INPUT PEKT
67 NEXT K
68 FOR K=1 TO L2
69 DISP "ENTER THETH(";K;")";
70 INPUT TEK1
71 NEXT K
72 DISP "ENTER COEF
                                TOLERENCE";
73 INPUT D2
74 DISP "ENTER W BAR";
75 INPUT W1
76 L3=P1
77 IF Q1<P1 THEN 80
78 L3=Q1
80 FOR k=1 TO L3
85 DISP "ENTER B(";K;")";
90 INPUT B(K)
95 NEXT K
100 FOR 1=N TO 1 STEP -1
110 W[ I+L3 ]=W[ I ]
120 NEXT I
250 P5=0.5
260 GOSUB 3000
270 Z=S1
280 FOR J=1 TO N+L3
290 X[ 1:11]=A[ J]
300 NEXT J
302 M=0
305 IF W1=0 THEN 410
306 M=1
310 Z1=W1
320 W1=Z1*1.00001
330 GOSUB 3000
340 FOR J=1 TO N+L3
350 IF W1=0 THEN 380
360 X[J,1]=(X[J,11]-A[J])/(0.00001+Z1)
370 GOTO 390
380 X[ J, 1 ]=0
385 NEXT J
386 GOTO 410
390 W1=Z1
400 NEXT J
410 FOR L=1 TO L1
420 Z1=P[L]
430 PCL J=Z1*1.00001
440 GOSUB 3000
```

TABLE B-II. (CONTINUED)

```
450 FOR J=1 TO N+L3
460 X[J,L+M]=(X[J,11]-A[J])/(0.00001*ZI)
470 NEXT J
480 P[L]=Z1
490 NEXT L
495 Z2=L1
500 FOR L=1 TO L2
510 Z1=T[L]
520 T[L]=1.00001+Z1
530 GOSUB 3000
540 FOR J=1 TO N+L3
550 X[J,L+M+L1]=(X[J,11]-A[J])/(0.00001*Z1)
560 NEXT J
570 T[L]=21
580 NEXT L
590 FOR L=1 TO P1
595 Z1=B[L]
600 IF Z1=0 THEN 626
605 B[L]=Z1*1.00001
610 GOSUB 3000
615 FOR J=1 TO N+L3
620 X[J,L+M+L1+L2]=(X[J,11]-A[J])/(0.00001*Z1)
622 NEXT J
624 GOTO 636
626 BCL ]=Z1+0.00001
628 GOSUB 3000
630 FQR J=1 TO N+L3
632 X[J,L+M+L1+L2]=(X[J,11]-A[J])/0.00001
634 NEXT J
636 B[L]=Z1
638 NEXT L
640 Z1=M+L1+L2+P1
650 REDIM VCZ1, Z1 ], GCZ1 ], HCZ1 ], UCZ1, Z1 ], RCZ1, Z1 ]
670 FOR I=1 TO Z1
680 G[ I ]=0
690 FOR J=1 TO Z1
700 Y[I,J]=0
710 FOR K=1 TO N+L3
720 YEI, J]=YEI, J]+XEK, I]*XEK, J]
730 NEXT K
740 NEXT J
750 FOR K=1 TO N+L3
760 G[ [ ]=G[ [ ]+X[ K, [ ]*X[ K, [ ] ]
770 NEXT K
780 NEXT I
790 FOR J=2 TO Z1
800 FOR J=1 TO J
810 YEJ, I J=YEI, J J
820 NEXT I
830 NEXT J
835 MAT PRINT Y
836 MAT PRINT G
970 MAT Y=INV(Y)
980 MAT H=Y*G
1010 MAT PRINT H
1015 IF W1=0 THEN 1025
1020 W1=W1+H[1]
1022 PRINT "MEAN="; W1
1025 FOR I=1 TO L1
```

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TABLE B-II. (CONTINUED)

```
1030 P[[]=P[]]+H[]+H]

1035 PRINT "FHI(";I;")=";P[]]

1040 NEXT I

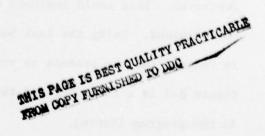
1050 FOR I=1 TO L2

1060 T[]=T[]]+H[]2+I+M]

1065 PRINT "THETA(";I;")=";T[]]
1070 NEXT I
1080 FOR I=1 TO F1
1090 BCI ]=BCI ]+HCL1+L2+M+I ]
1095 PRINT "N("; 1; ")="; BCI ]
1100 NEXT I
1110 GOSUB 3000
1115 IF S1 <= Z THEN 1130
1117 GOSUB 2000
1120 MAT H=(0.5)*H
1125 GOTO 1010
1130 T1=0
1140 FOR I=1 TO Z1
1150 IF ABS(H[1])(D2 THEN 1170
1160 T1=T1+1
1170 NEXT 1
1180 IF T1=0 THEN 1320
1185 P5=P5/2
1230 G0T0 270
1330 $2=$1/(H-P1)
1340 MAT U=TRN(Y)
1350 MAT R=U+Y
1360 MAT Y=INV(R)
1370 PRINT "RESIDUAL VARIANCE=";52
1380 PRINT "LEAST SQUARES ESTIMATES OF PARAMETERS ARE:"
1390 PRINT "MEAN=";WI
1400 FOR 1=1 TO L1
1410 PRINT "PHI("; I; ")=";P[I]
1420 NEXT I
1430 FOR I=1 TO L2
1440 PRINT "THETA(": I:")=";T[]]
1450 NEXT 1
1452 FOR I=1 TO P1
1454 PRINT "N(";I;")=";B[]]
1456 NEXT I
1460 MAT Y=(S2)*Y
1470 PRINT "COVARIANCE MATRIX IS"
1480 MAT PRINT Y
1481 FOR J=1 TO N+L3
1482 ACJ]=ACJ+L3]
1483 NEXT J
1490 DISP "ENTER FILE NR FOR RESIDUAL":
1500 INPUT A2
1510 STORE DATA AZ.A
1520 STOP
2000 W1=W1-H[1]
2010 FOR I=1 TO L1
2020 P[ ] =P[ ] ]-H[ ]+N ]
2030 NEXT I
2040 FOR I=1 TO L2
2050 T[]=T[]-H[Z2+[+M]
2060 NEXT I
2070 FOR I=1 TO P1
```

TABLE B-II. (CONCLUDED)

```
2080 B[I]=B[[]-H[L1+L2+N+I]
2090 NEXT I
2100 PETURN
3000 FOR I=1 TO L3
3040 A[I]=0
3054 W[I]=B[I]
3056 NEXT I
3240 S1=0
3250 FOR J=1+L3 TO N+L3
3260 A[J]=FNA(J)
3270 S1=S1+A[J]*A[J]
3280 NEXT J
3290 PRINT "SUM SQUARES=";S1
3300 RETURN
3310 STOP
4240 END
5000 DEF FNA(T)=WCT]-P[I]*W[T-1]-P[2]*WCT-2]-W1*(1-P[I]-P[2])*T[I]*A[T-1]
```



base point, \underline{b}_1 . Define the vector $\underline{t}_{i,j}$, called the temporary head, where the subscript i denotes the number of the temporary head and the subscript j denotes the element of the parameter vector, \underline{t} , that is being perturbed. The vector $\underline{t}_{i,j}$ defines a point in parameter space about which exploration is taking place. Let δ_j be some step size associated with the parameter ξ_j . Let the first temporary head be the base point, i.e., let $\underline{t}_{1,0} = \underline{b}_1$. Assume $\underline{\xi}$ is of dimension m. Perturb each of the elements of the parameter vector, one at a time, so that $j = 1, 2, \ldots$ m. Calculate a new temporary head for each variable such that the new $\underline{t}_{i,j}$ is that vector which yields the minimum of $\{S(t_{i,j-1}) + \delta_j\}$, $\{(t_{i,j-1}) - \delta_j\}$, $\{(t_{i,j-1}) \}$. Note that each temporary head makes use of the best value resulting from the last parameter to be perturbed.

When all the variables have been perturbed, $\underline{\mathbf{t}}_{1,m}$ is designated as a second base point, $\underline{\mathbf{b}}_2$. It will be the best result from all the perturbations up to this point. The vectors $\underline{\mathbf{b}}_1$ and $\underline{\mathbf{b}}_2$ establish the first pattern. A new temporary head is established at a point on the vector between $\underline{\mathbf{b}}_1$ and $\underline{\mathbf{b}}_2$ and twice the distance between them. The process is then repeated at this new temporary head. If at any iteration the temporary head $\mathbf{t}_{i,m}$ is no better than the last base point the pattern is destroyed. This could indicate either a ridge or that an optimum has been reached. Using the last base point, the perturbation step size is reduced and the process is repeated until it converges to a solution. Figure B-3 is a flow chart of the pattern search algorithm; Table B-III is the program listing.

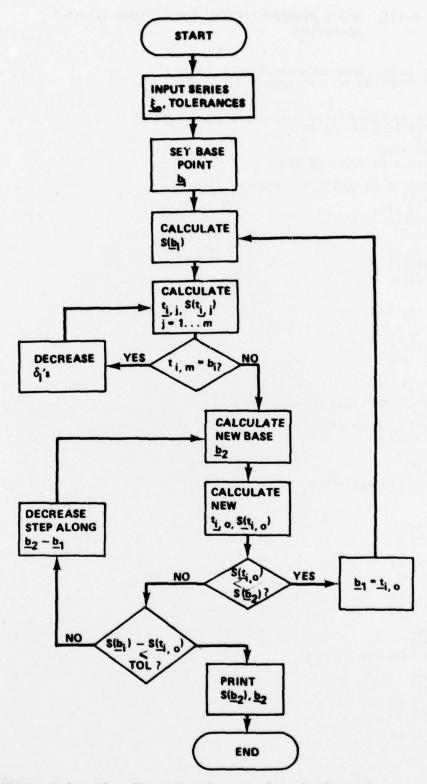


Figure B-3. Flow Chart For Pattern Search Algorithm.

TABLE B-III. BASIC PROGRAM LISTING FOR PATTERN SEARCH ALGORITHM

```
10 REM CONDITIONAL LEAST SQUARES PATTERN SEARCH 20 DISP DEF FUNCTION A(T) AT 5000:
30 STOP
35 D=0.01
40 DIM ASI 200]. NSI 200], PSI 10, 3], TSI 10, 3], MS[ 3], BSI 10, 3]
50 DISP
           "ENTER FILE NR, NR PTS";
60 INPUT AL.N
          DATA A1.W
"ENTER # AR COEFS, AR ORDER";
70 LOAD
80 DISP
90 INPUT L1.P1
100 DISP "ENTER # MA COEFS, MA ORDER":
110 INPUT L2,01
120 FOR I=1 TO L1
130 DISP "ENTER P(":1:")";
140 INPUT PEI.11
150 NEXT I
160 FOR I=1 TO L2
170 DISP "ENTER T("; [;")";
180 INPUT T[ 1.1]
190 NEXT I
200 Q3=P1
210 IF P1 >= 01 THEN 230
220 03=01
230 FOR I=1 TO Q3
240 DISP "ENTER B(";[;")";
250 INPUT B[[;1]
260 NEXT :
265 DISP "ENTER MEAN":
267 INPUT ME11
268 DISP "ENTER COEF: SUM SQUARES TOL":
269 INPUT D1.D2
270 FOR J=N+03 TO Q3+1 STEP -1
280 W[J]=W[J-Q3]
290 NEXT
300 GOSUB 3000
312 GOSUB 1060
315 PRINT "SUM SQUARES ="\$1
320 Z2=$1
630 Z3=Z2
640 M[2]=N[1]
650 FOR I=1 TO 03
660 B[1,2]=B[1,1]
670 NEXT I
680 FOR I=1 TO L1
690 PLI-23=PLI-13
700 NEXT
 710 FOR I=1 TO L2
720 T[ 1 - 2 1= T[ 1 - 1 ]
730 NEXT I
740 COSUB 3560
750 GOSUB 3000
760 IF S1 23 THEN 890
765 22≈23
766 609UP 1060
780 ME 2 ]=ME 3 ]
790 FOR I=1 TO 03
800 B[ I + 2 ]=B[ I + 3 ]
810 NEXT I
820 FOR I=1 TO L1
```

TABLE B-III. (CONTINUED)

```
830 P[ 1.2]=P[ 1.3]
840 NEXT I
850 FOR I=1 TO L2
860 T[ 1,2]=T[ 1.3]
870 NEXT I
875 PRINT "SS="123;"D="1D
880 GOTO 740
890 GOSUB 1060
900 PRINT "SUM SQUARES=";S1
910 IF D)D1 THEN 920
914 IF ABS(Z2-Z3) <= D2 THEN 930
920 GOTO 320
930 PRINT "LOCAL MINIMUM REACHED"
940 DISP "ENTER FILE NR FOR RESIDUALS":
942 FOR J=1 TO N
944 A[J]=A[J+Q3]
946 NEXT J
950 INPUT A2
960 STORE DATA AZ.A
970 STOP
1060 PRINT "MEAN="; M[1]
1070 FOR [=1 TO Q3
1080 PRINT "B(";[;")=";B[[,1]
1090 NEXT I
1100 FOR I=1 TO L1
1110 PRINT "P(": I;")=";P[I,1]
1120 NEXT I
1130 FOR I=1 TO L2
1140 PRINT "T(";I;")=";T[I,1]
1150 NEXT |
1155 RETURN
3000 FOR J=1 TO 03
3010 A[J]=0
3020 WEJ ]=BE J, 11
3030 NEXT J
3040 S1=0
3050 FOR J=Q3+1 TO N+Q3
3060 ALJ]=FNA(J)
3070 S1=S1+ALJ]†2
3080 NEXT J
3090 RETURN
3100 STOP
3500 REM PATTERN SEARCH
3510 M[1]=M[2]+D
3520 GOSUB 3000
3525 PRINT M[1],S1
3530 IF S1<Z3 THEN 3560
3540 M[3]=M[2]
3550 GOTO 3580
3560 Z3=81
3570 ME 3 1=ME 1 1
3580 ME 1 J=ME 2 J-D
3590 GOSUB 3000
3595 PRINT MC1],S1
3600 IF St<Z3 THEN 3630
3610 MC1 ]=MC3]
3620 GOTO 3650
3630 Z3=S1
3640 ME 3 ]=ME ! ]
```

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TABLE B-III. (CONTINUED)

```
3650 FOR I=1 TO 03
 3660 B[ I . 1 ]=B[ I . 2 ]+D
 3670 GOSUB 3000
3675 PRINT B[1,1],S1
3680 IF S1<23 THEN 3710
3690 B[1,3]=B[1,2]
3700 GOTU 3730
 3710 B[1.3]=B[1.1]
3720 23=$1
3730 B[1.1]=B[1.2]-D
3740 Bt I 1 1 = Bt 1 2 1 - B

3740 GOSUB 3888

3745 PR SI 23 THEN 3788

3750 Bt I 1 1 = Bt I 1 3 1

3770 GOTO 3838

3780 Bt I 2 3 = Bt I 1 1

3790 Z3 = S1

3880 Bt Y I
 3800 NEXT I
  3810 FOR I=1 TO L1
 3820 P[1.1]=P[1.2]+D
 3830 GOSUB 3000
3835 PRINT PLI.11.51
3840 IF SI<23 THEN 3870
 3850 P[1.3]=P[1.2]
 3860 GOTO 3890
 3870 Z3=S1
 3880 P[[,3]=P[[,1]
 3890 P[[.1]=P[[.2]-D
3890 P[[:1]=P[[:2]-D]
3900 GOSUB 3000
3905 PRINT P[[:1],S1
3910 IF SICZ3 THEN 3940
3920 P[[:1]=P[[:3]]
3930 GOTO 3960
3940 P[[:3]=P[[:1]]
3950 Z3=S1
 3960 NEXT
 3970 FOR I=1 TO L2
 3980 T[[,1]=T[[,2]+D
3990 GOSUB 3000
3995 PRINT TEL.IJ.SI
4000 IF SI<23 THEN 4030
4010 TEL.3J=TEL.21
4020 GOTU 4050
4030 Z3=S1
4040 T[1,3]=T[1,1]
4050 T[ [ 1 ]=T[ [ 2]-D
4066 GOSUB 3888
4065 PRINT TEL:13:S1
4070 IF S1:23 THEN 4888
4074 TEL:13=TEL:33
4076 GOTO 4100
4080 TEL:33=TEL:13
4090 Z3=S1
4100 NEXT I
4110 IF Z3<Z2 THEN 4140
4114 IF D:01 THEN 4120
4116 IF ABS(Z2-Z3) <= D2 THEN 930
4120 D=D+0.6
4122 GOSUB 1060
```

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TABLE B-III. (CONCLUDED)

```
4124 PRINT "SS="; Z3, Z2
4130 GOTO 3500
4140 M[1]=2*M[3]-M[2]
4150 FOR I=1 TO Q3
4160 B[I,1]=2*B[I,3]-B[I,2]
4170 NEXT I
4180 FOR I=1 TO L1
4190 P[I,1]=2*P[I,3]-P[I,2]
4200 NEXT I
4210 FOR I=1 TO L2
4220 T[I,1]=2*T[I,3]-T[I,2]
4230 NEXT I
4240 RETURN
4250 STOP
4260 END
5000 DEF FNA(T)=WEI]-P[1,1]*WEI-1]-P[2,1]*WEI-2]-MEI]*(1-P[1,1]-P[2,1])
```

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